Simulation and analysis of polarized synchrotron emission maps

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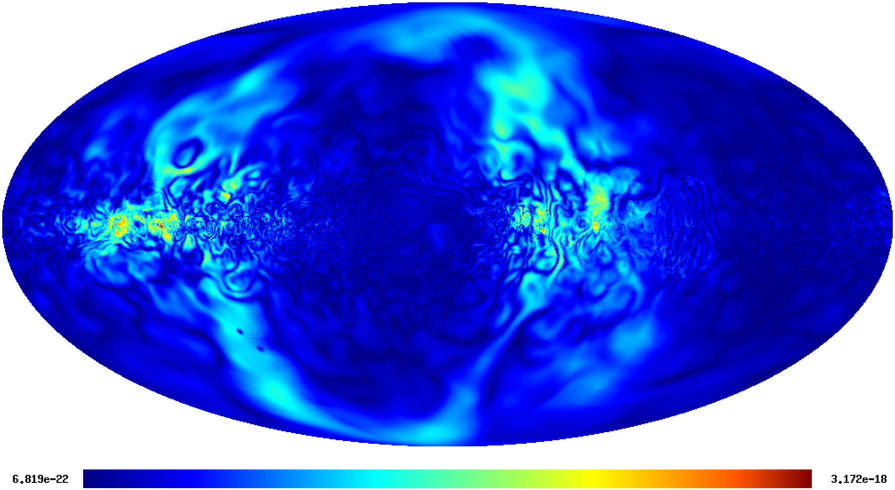
turbulence in the magnetized ISM

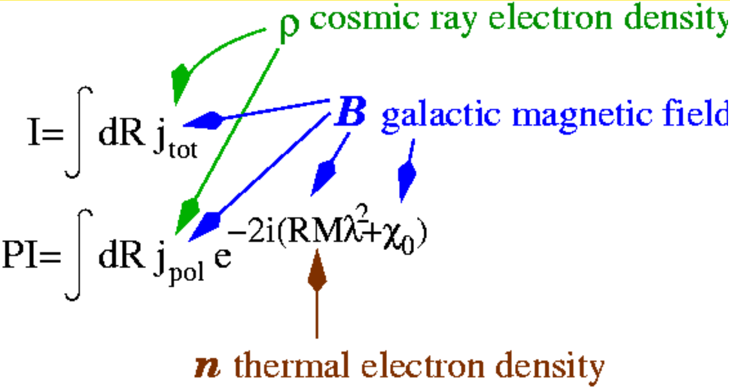
outline

1. Hammurabi, the galactic synchrotron sky simulation code
2. Stokes-correlators, analysing PI maps

Part 1: The Hammurabi code



What we get!



Ingredients:

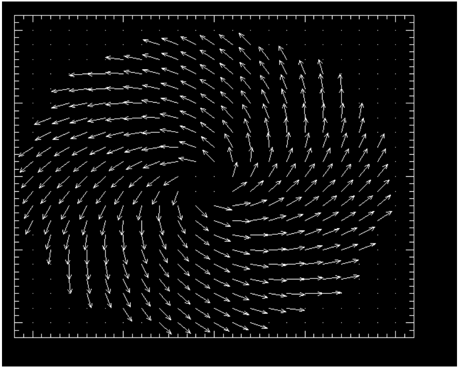
- thermal electron density model (NE2001, Cordes & Lazio 2002)
- cosmic ray electron density (WMAP3, GALPROP?, our own)
- magnetic field models (several)

Compilation of input models

- WMAP 3th year (Page et al. 2006) ← this talk!
  - magnetic field
  - cosmic ray electron distribution
- Prouza and Smida 2003
  - magnetic field
- Stanev 1987
  - magnetic field
- and others...

regular magnetic field – wmap-3 model

$$\vec{B}(r,\phi,z) = B_0[\cos\psi(r)\,\cos\chi(z)\hat{r} + \sin\psi(r)\,\cos\chi(z)\hat{\phi} + \sin\chi(z)\hat{z}]$$



cosmic ray electrons – wmap-3 model

power-law energy distribution in an exponential disk:

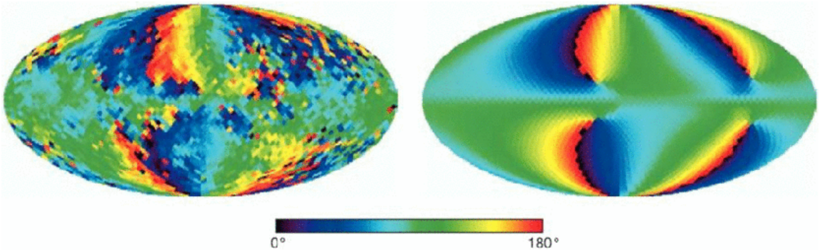
$$n_e = n_0 \exp(-r/h_r)\,\mathrm{sech}^2(z/h_d).$$

scale height: $h_d = 1$ kpc
radial scale length: $h_r = 5$ kpc
n_0 unspecified

Page et al. (2006)

WMAP 3

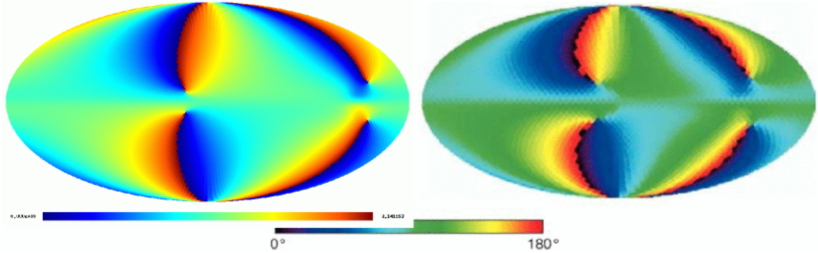
polarization direction



Page et al. 2006

WMAP 3

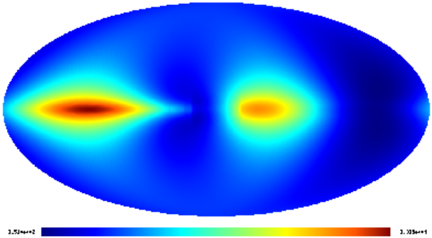
polarization direction



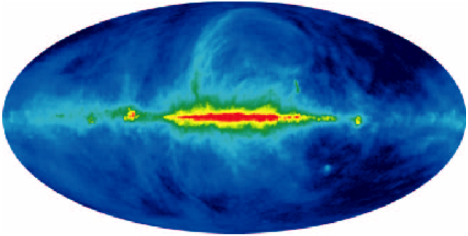
Page et al. 2006

WMAP 3

I maps



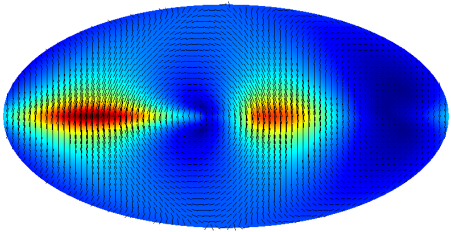
Hammurabi

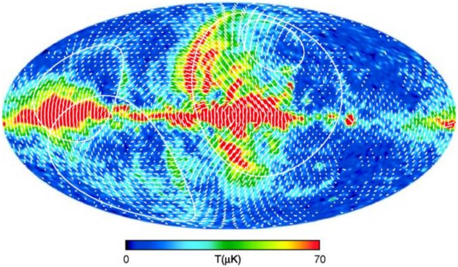


Haslam et al. 1982 (from Bennett et al. 2003)

WMAP 3

PI maps

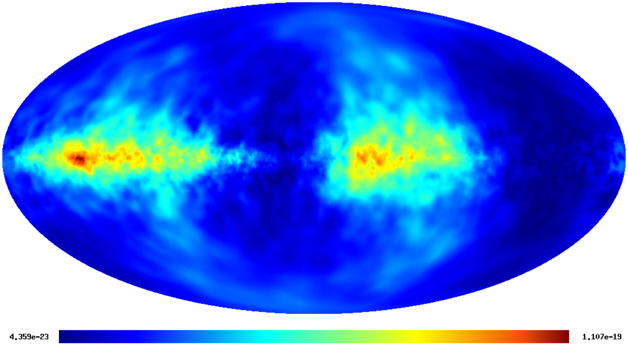




Page et al. 2006

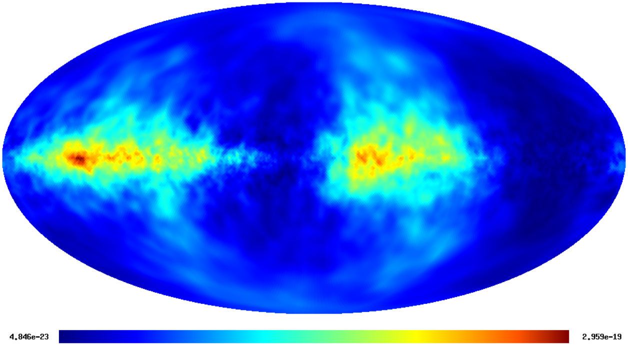
Faraday depolarization

60 GHz



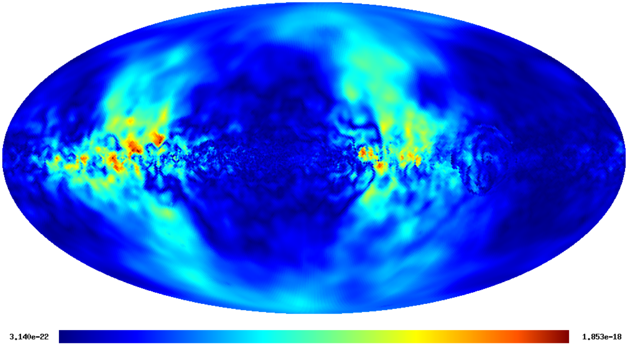
Faraday depolarization

22 GHz



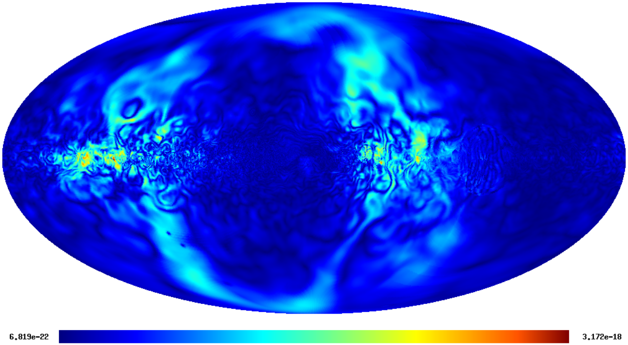
Faraday depolarization

2.4 GHz



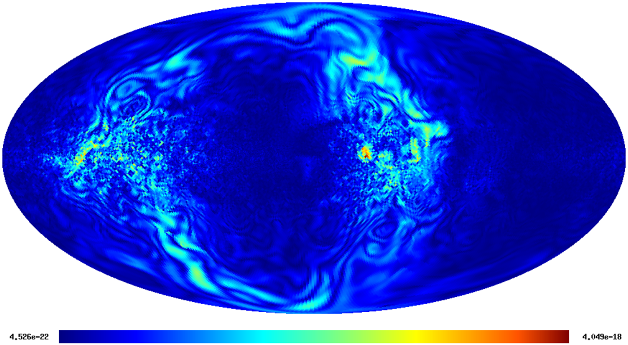
Faraday depolarization

1.4 GHz



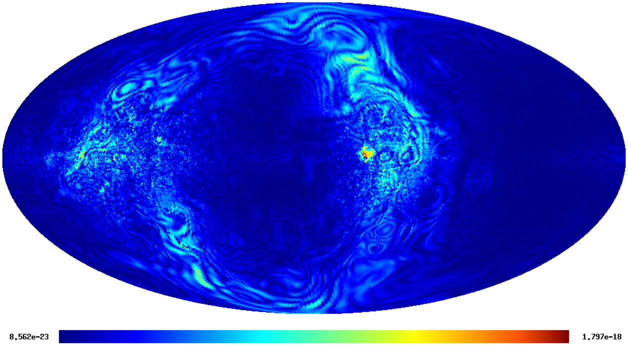
Faraday depolarization

0.8 GHz



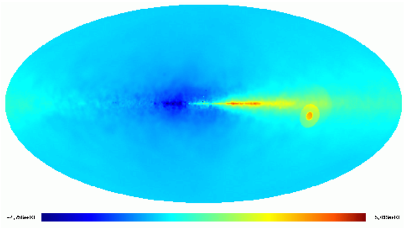
Faraday depolarization

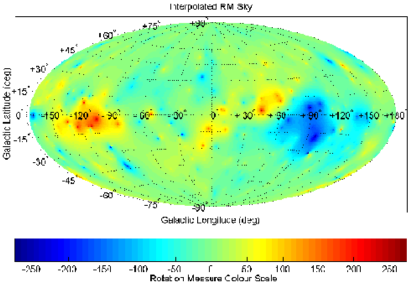
0.4 GHz



WMAP 3

RM maps

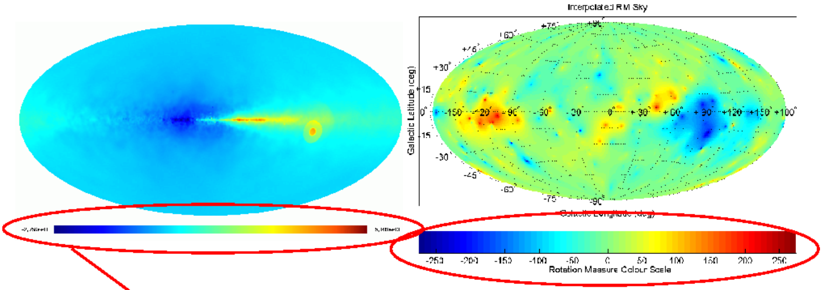




Johnston-Hollitt et al. 2004

WMAP 3

Excessive RM!

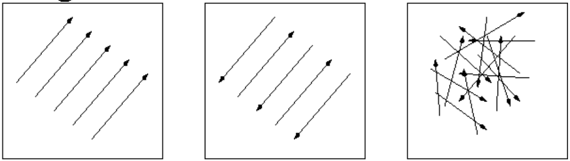


one order of magnitude larger then:

Johnston-Hollitt et al. 2004

Our approach

Maximum code resolution is a $\sim$ cubic cell of $\sim 64\mathrm{pc}^3$.
We need subgrid modeling:



	regular	ordered	random
I	✓	✓	✓
PI	✓	✓	✗
RM	✓	✗	✗

Part 2: Analysing polarized intensity maps

We would like to see whether there is information about the magnetic field encrypted in PI maps.

What is the scenario?

We hope to be able to find somewhere something like this:

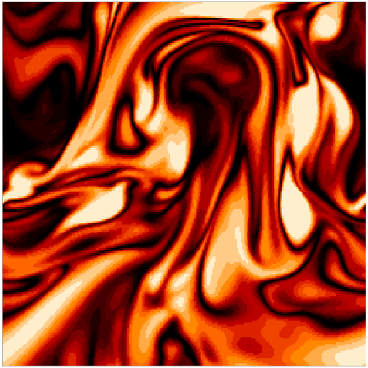


Figure: Cross section of the field strength in the saturated state of a simulation of homogeneous isotropic MHD turbulence (run B in Schekochihin et al 2004)

A first explorative attempt

We propose to investigate the power spectrum of the Lorentz force.

$$\frac{1}{c}(\vec{J} \times \vec{B}) = -\nabla(\frac{B^2}{8\pi}) + \frac{1}{4\pi}\left[(\vec{B}\nabla)\vec{B}\right]$$

In particular (the magnetic tension-force)

$$\vec{F} = \frac{1}{4\pi}\left[(\vec{B}\nabla)\vec{B}\right]$$

A first explorative attempt

magnetic tension-force

$$\vec{F} = \frac{1}{4\pi}\left[(\vec{B}\nabla)\vec{B}\right]$$

"tension-force" correlations:

$$\hat{\Phi}_{im}(\vec{k}) =< \hat{F}_i(\vec{k})\hat{F}_m(\vec{k}') >= (2\pi)^3\delta(\vec{k}+\vec{k}')k_j k_n \hat{C}_{ij,mn}(\vec{k}')$$

Where,

$$C_{ij,mn}(\vec{r}) =< B_i(\vec{x}_1)B_j(\vec{x}_1)B_m(\vec{x}_2)B_n(\vec{x}_2) >$$

and,

$$r = |\vec{r}| = |\vec{x}_1 - \vec{x}_2|$$

A first explorative attempt

Originaly there are 26 functions, simplified by symmetries to seven unknown:

$$
\begin{aligned}
\hat{C}_{ij,mn}(k) \quad = \quad & \hat{C}_1(k)\delta_{ij}\delta_{mn} + \hat{C}_2(k)(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}) \\
& + \hat{C}_3(k)(\delta_{ij}\hat{k}_m\hat{k}_n + \delta_{mn}\hat{k}_i\hat{k}_j) \\
& + \hat{C}_4(k)(\delta_{im}\hat{k}_j\hat{k}_n + \delta_{in}\hat{k}_j\hat{k}_m + \delta_{jm}\hat{k}_i\hat{k}_n + \delta_{jn}\hat{k}_i\hat{k}_m) \\
& + \hat{C}_5(k)\hat{k}_i\hat{k}_j\hat{k}_m\hat{k}_n \\
& + \hat{C}_6(k)(\epsilon_{imp}\hat{k}_p\delta_{jn} + \epsilon_{inp}\hat{k}_p\delta_{jm} + \epsilon_{jmp}\hat{k}_p\delta_{in} + \epsilon_{jnp}\hat{k}_p\delta_{im}) \\
& + \hat{C}_7(k)(\epsilon_{imp}\hat{k}_p\hat{k}_j\hat{k}_n + \epsilon_{inp}\hat{k}_p\hat{k}_j\hat{k}_m + \epsilon_{jmp}\hat{k}_p\hat{k}_i\hat{k}_n + \epsilon_{jnp}\hat{k}_p\hat{k}_i\hat{k}_m)
\end{aligned}
$$

Observables: Stokes parameter:

$$
\begin{aligned}
I(\vec{x}_\perp) &= \int_{z_0}^{\infty} dz\,(\delta B_x^2(\vec{x}) + \delta B_y^2(\vec{x})) \\
Q(\vec{x}_\perp) &= \int_{z_0}^{\infty} dz\,(\delta B_x^2(\vec{x}) - \delta B_y^2(\vec{x})) \\
U(\vec{x}_\perp) &= \int_{z_0}^{\infty} dz\,(\delta B_x(\vec{x})\delta B_y(\vec{x}))
\end{aligned}
$$

... and their six correlation functions (Notice! It is a projection!):

$$\Sigma_{II}(\vec{r}_\perp), \Sigma_{QQ}(\vec{r}_\perp), \Sigma_{UU}(\vec{r}_\perp), \Sigma_{IQ}(\vec{r}_\perp), \Sigma_{IU}(\vec{r}_\perp), \Sigma_{QU}(\vec{r}_\perp)$$

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$$

The symmetric part of the tension-force correlation tensor is fully encoded in the data, i.e. the tension-force power spectrum is:

$$\hat{\Phi}_{ii}(\vec{k}_\perp, 0) = \frac{1}{4}k^2 \left[\hat{\Sigma}_{II}(\vec{k}_\perp) + \hat{\Sigma}_{QI}(\vec{k}_\perp) + \hat{\Sigma}_{IQ}(\vec{k}_\perp) + \hat{\Sigma}_{QQ}(\vec{k}_\perp) + 8\hat{\Sigma}_{UU}(\vec{k}_\perp)\right]$$

This demonstrates that the method obtains physically relevant information on MHD turbulence from polarization data!

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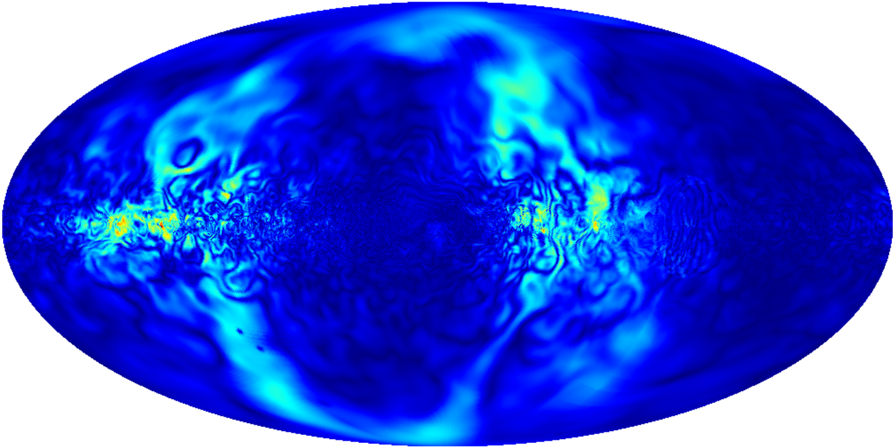
$$\begin{aligned}\hat{\Phi}_{ii}(\vec{k}_\perp, 0) &= \frac{1}{4}k^2 \left[\hat{\Sigma}_{II}(\vec{k}_\perp) + \hat{\Sigma}_{QI}(\vec{k}_\perp)\right. \\ &\quad \left.+\hat{\Sigma}_{IQ}(\vec{k}_\perp) + \hat{\Sigma}_{QQ}(\vec{k}_\perp) + 8\hat{\Sigma}_{UU}(\vec{k}_\perp)\right]\end{aligned}$$

This demonstrates that the method obtains physically relevant information on MHD turbulence from polarization data!

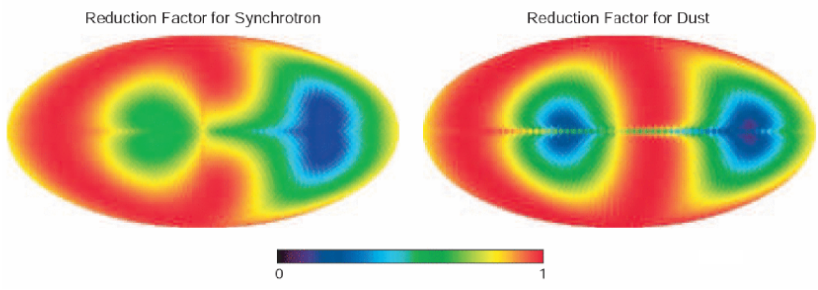
summary

- synchrotron sky simulation tool (Hammurabi code)
  - small scale magnetic field requires more sofisticated modeling
- Stokes correlation method
  - there is physicaly relevant information encoded in PI maps
  - we will be looking fro scenario that accomodates our assumptions in the intra cluster medium.

Thank you!



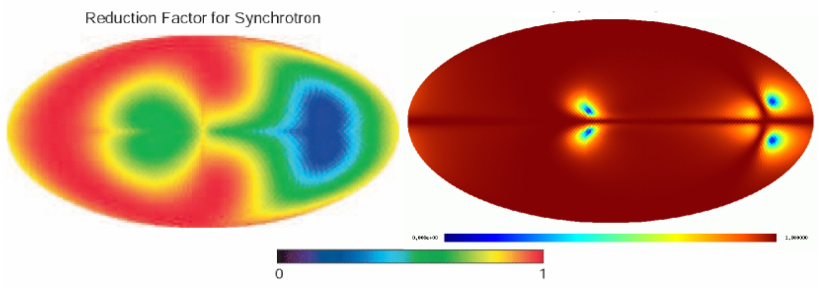
depolarisation: synchrotron vs. dust



$g_{sync}(\hat{n}) = \frac{P(\hat{n})}{\Pi_s \;\; I(\hat{n})}$ (geometric suppression)

$\Pi_s = (p+1)/(p+7/3) \approx 0.75$ (intrinsic polarization fraction)
$P(\hat{n}) = \sqrt{Q^2 + U^2}$ & $I(\hat{n})$ from LOS integration

depolarisation: wmap calc. vs. our calc.



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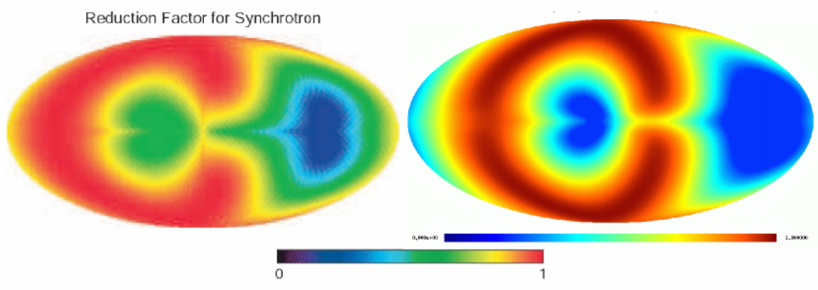
depolarisation: wmap calc. vs. our calc. + wrong $I(\hat{n})$



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